Barem de corectare OLM 2019 Clasa a XI-a

P1

$-1=\lim\limits_{x\to-\infty} x\left(a-\sqrt{b+\frac{c}{x}-\frac{1}{x^2}}\right)$ **;** **î**n mod necesar $a=\sqrt{b}$	1p
$-1=\lim\limits_{x\to-\infty}\left(\sqrt{b}x+\sqrt{bx^2+cx-1}\right)=\lim\limits_{x\to-\infty}\frac{-cx+1}{x\left(\sqrt{b}+\sqrt{b+\frac{c}{x}-\frac{1}{x^2}}\right)}=-\frac{c}{2\sqrt{b}}\Rightarrow c=2\sqrt{b}$	2p
$2=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{\sqrt{b}x+\sqrt{bx^2+cx-1}}{x}=2\sqrt{b}\Rightarrow b=1, c=2, a=1$	3p
Se verifică valorile găsite $1=\lim\limits_{x\to\infty}\left(f(x)-2x\right)=\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x-1}-x\right)=\lim\limits_{x\to\infty}\frac{2x-1}{\sqrt{x^2+2x-1}+x}=1$.	1p

P2

a) $\det(A)=1\Rightarrow\det\left(A^{2019}\right)=\left(\det(A)\right)^{2019}=1$	3p
b) Folosind relația Cayley-Hamilton $A^2-tr(A)\cdot A+\det(A)\cdot I_2=O_2$, se obține $A^2=A-I_2$	3p
Înmulțind egalitatea cu A^n se obține relația cerută.	1p

P3 – autor Doru Isac

a) $X=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\in M_2(Z)$. $XA=AX\Leftrightarrow\begin{pmatrix}3a-b & 6a-2b\\ 3c-d & 6c-2d\end{pmatrix}=\begin{pmatrix}3a+6c & 3b+6d\\ -a-2c & -b-2d\end{pmatrix}$	2p
$\Rightarrow X=\begin{pmatrix}a & -6c\\ c & a+5c\end{pmatrix}, a,c\in Z$	1p
b) $X^{2020}=X\cdot A=A\cdot X\Rightarrow X=\begin{pmatrix}a & -6c\\ c & a+5c\end{pmatrix}$	1p
$\det(A)=0\Rightarrow\det(X)=0\Rightarrow a^2+5ac+6c^2=0\Rightarrow a\in\{-3c,-2c\}$	1p
$X=\begin{pmatrix}-3c & -6c\\ c & 2c\end{pmatrix}=-c\cdot A$ sau $X=\begin{pmatrix}-2c & -6c\\ c & 3c\end{pmatrix}=c\cdot B$, $B=\begin{pmatrix}-2 & -6\\ 1 & 3\end{pmatrix}$ $a=-2c$, $X^{2019}=c^{2019}\cdot B^{2019}=c^{2019}\cdot B$, dar $c^{2019}\cdot B=A$, imposibil	1p
$a=-3c$, $X^{2019}=(-c)^{2019}\cdot A^{2019}=(-c)^{2019}\cdot A\Rightarrow c=-1\Rightarrow X=A$, soluție unică	1p

P4 – autor Benedict G. Niculescu (GM 10/2018)

$L_1=\lim\limits_{n\to\infty}\frac{1+\sqrt[4]{2}+\sqrt[4]{3}+...+\sqrt[4]{n}}{n\sqrt[4]{n}}$ și $L_2=\lim\limits_{n\to\infty}\sqrt[4]{n^2}\left(\sqrt[4]{(n+1)^2+1}-\sqrt[4]{n^2+1}\right)$	1p
Pentru L_1 folosim lema lui Stolz $L_1=\lim\limits_{n\to\infty}\frac{\sqrt[4]{n+1}}{(n+1)\sqrt[4]{n+1}-n\sqrt[4]{n}}=\lim\limits_{n\to\infty}\frac{\sqrt[4]{n+1}}{n\sqrt[4]{n}\left[\left(\frac{n+1}{n}\right)^{1+\frac{1}{4}}-1\right]}=\lim\limits_{n\to\infty}\frac{\sqrt[4]{\frac{n+1}{n}}}{\frac{\left(1+\frac{1}{n}\right)^{\frac{5}{4}}-1}{\frac{1}{n}}}=\frac{4}{5}$	3p
$L_2=\lim\limits_{n\to\infty}\sqrt[4]{n^4+n^2}\left(\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}-1\right)=\lim\limits_{n\to\infty}\frac{\left(1+\frac{2n+1}{n^2+1}\right)^{\frac{1}{4}}-1}{\frac{2n+1}{n^2+1}}\cdot\frac{\sqrt[4]{n^4+n^2}\,(2n+1)}{n^2+1}=\frac{1}{2}$	2p

$L = \frac{L_1}{L_2} = \frac{\frac{4}{5}}{\frac{1}{2}} = \frac{8}{5}$	1p